Instability Suppression

C. Carli
CERN/PS, CH 1211 Genève 23

Various instabilities hamper the operation of accelerators and degrade the quality of particle beams. These include single-particle phenomena, like the excitation of resonances, and collective instabilities. Ways of fighting these instabilities will be reviewed with a particular emphasis on cooling rings.

1 Introduction

The emittances, which can reached in synchrotrons equipped with cooling systems are small, and thus even beams with moderate intensities become vulnerable for space charge effects. Instabilities due to space charge effects are important limitations for synchrotrons equipped with cooling systems and this is the topic of various review articles [1, 2]. The limiting effects considered in this paper are :

- The direct space-charge effect becomes important due to the small beam sizes. This leads to a large Laslett tune shift and strong additional nonlinear forces. Thus, the single-particle equations are effected and the oscillation amplitude may increase due to resonant behaviour, with the result that the particle may be lost. In many cases, a cooling system has a stabilizing effect.
- Due to the small emittances in all three phase planes, the spreads in oscillation frequency become small. Thus, Landau damping becomes weak and collective instabilities often occur. A cooling system may contribute to the excitation of instabilities, but may also have a small stabilizing effect.

These instabilities and their cures are reviewed with emphasis on cooling rings.

2 Resonances

Particles in a synchrotron see not only the linear focusing forces, but also nonlinear components. This can lead to an increase of the transverse oscillation amplitudes and to beam loss, due to a resonant behaviour. Thus, the working point of a synchrotron must be choosen appropriately, with sufficient distance to dangereous low-order resonances. However, due to space-charge effects, not all particles oscillate with the same tunes, but there is a spread in tune, which is a sizable fraction of the Laslett tune shift describing the tune shift for particles with small oscillation amplitudes. For a rough estimate, the modulation of the beam size around ring via the betatron function is neglected and the beam is assumed round. Then the Laslett tune shift is given by :

$$\Delta Q = -\frac{Nq^2}{8\pi^2\epsilon_0 E_0} \cdot \frac{F}{\beta^2\gamma^3\epsilon}$$

for a coasting beam (a derivation can be found in references [3, 4]). The meaning of the symbols used is explained in appendix A. In cooler rings, the Laslett tune shift can reach high values even with moderate intensities, due to the small emittances reached. Thus it becomes difficult to locate the working point such that all particles stay away from resonances. Furthermore, the direct space-charge not only spreads out the tunes, but is also highly nonlinear and thus excites resonances. Experience shows that the maximum acceptable Laslett tune shift depends strongly on the time the beam stays in a machine and on whether a cooling system is present. For rings equipped with an electron cooler, Laslett tune shifts of about 0.2 have been observed.

Another source of space-charge forces comes from the electron cooler, especially if the beam to be cooled has larger transverse dimensions than the electron beam. Then, the additional force due to the potential induced by the electron beam space charge is highly nonlinear at the edge of the electron beam and may contribute to the excitation of resonances. This effect is one of the hypothesis to explain the "electron heating" [5] observed at CELSIUS. But there are also attempts to explain this "electron heating" by collective effects.

Limitations due to single-particle resonances and the tune spread can be reduced by :

- Careful choice of the working point in a region sufficiently far from dangereous low-order resonances.
- Limitation of the resonance excitation coefficients. This can be done by careful construction of the accelerator magnets and by compensation of the resonance excitation coefficients. In the case of a ring equipped with an electron cooler, the transverse dimensions of the electron beam should be larger than the size of the circulating beam.
- Compensation of the amplitude increase due to resonances by a decrease due to the cooling system. This is the reason, why the acceptable Laslett tune shift is relatively large in accelerators equipped with a cooling system.

3 Collective Instabilities

3.1 Impedances

When the charge of the beam is not negligible, electromagnetic fields are excited by the beam passage. These fields depend on the surrounding of the beam, i.e. the properties of the vacuum chamber, equipments in vacuum tanks, etc. The electromagnetic fields react with the beam and may cause collective instabilities. The way, the fields created by the beam act back on the beam itself, is described by the longitudinal and transverse coupling impedance. Typical contributions to the total impedance of an accelerator are given in table 1.

Table 1: Main contributions to the total impedance of a cooling ring for a time dependence $e^{-i\Omega t}$.

Source of Impedance	$\frac{Z_{\|\|}/n}{\Omega}$	$\frac{Z_{\perp}/n}{\Omega/\mathrm{m}}$	Comment
Space charge	$i\frac{Z_0}{2\beta\gamma^2}\left(1+\ln\left(\frac{a}{b}\right)\right)$	$i\frac{Z_0 R}{\beta^2\gamma^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$	Circular chamber
Resistive wall	$(1-i)\frac{c\beta}{b}\sqrt{\frac{\mu_0}{2\Omega\sigma}}$	$(1-i)\frac{2cR}{b^3}\sqrt{\frac{\mu_0}{2\Omega\sigma}}$	Chamber thicker than skin depth
Kickers	$\approx \frac{l_K}{2\pi R} Z_0 \beta$	$\approx \frac{l_K}{2\pi}\cdot\frac{Z_0}{g_k h_k}$	Max. of real part, see ref. [7] l_K, g_k and h_K kicker length, gap height and width (total)
Cavities	$\frac{1}{n}\cdot\frac{1}{1+iQ_{cav}\left(\left(\frac{\omega_{cav}}{\Omega}-\frac{\Omega}{\omega_{cav}}\right)\right.}$	$\frac{2R}{b^2\beta}\frac{1}{1+iQ_{cav}\left(\frac{\omega_{cav}}{\Omega}-\frac{\Omega}{\omega_{cav}}\right)}$	
Electron Cooler	$\approx 2.2\frac{Z_0}{\beta^2}a_e\frac{l_c}{L}\sqrt{n_e r_e}$	$\approx 5.5\frac{Z_0}{\beta^2}\frac{R}{a_e}\frac{l_c}{L}\sqrt{n_e r_e}$	Max. of real part see ref. [8] At high frequencies

One way to fight instabilities is to reduce the impedances, and in particular to reduce their real part. However, "impedance hygiene" can reduce the impedance only to a certain level. Many rings applying electron cooling work at low energy and thus a large space-charge impedance is unavoidable. Furthermore, the cooling electron beam itself provides an additional field, which can also be described by coupling impedances. This can be the dominant contribution to the real part of the impedance. In the case of cooler rings using stochastic cooling, sensitive pick-ups are necessary in order to archieve a good signal-to-noise ratio, which implies a large impedance seen by the beam, at least inside the frequency band used for cooling.

3.2 Electron drift Instability

One instability mechanism, which cannot be described by the usual impedances, has been reported for rings with electron coolers. This is the electron drift instability [6, 9]. A transverse (say horizontal) displacement of the ion beam to be cooled leads to an additional transverse (horizontal) electric field acting on the electron beam. The electron beam drifts in the other (vertical) transverse direction, under the action of this electric field and the longitudinal magnetic field of the cooler. This in turn leads to an additional (vertical) kick on the ion beam. Since an offset in one plane leads to a deflection of the beam also in the other transverse plane, the electron drift effect cannot be described by the standard coupling impedances.

The effect becomes more complicated, if one takes into account that an offset in one plane leads to an offset in the other plane and that the ion beam may in addition pass the cooler with an angle. The electron drift effect is investigated by considering the 4×4 one-turn transfer matrices, which are not symplectic any more. Reference [6] comes to the conclusion that this effect does not lead to instability for positively charged particles and expects instability with growth rates in the order of 0.1 s to 10 s for antiprotons. Reference [9] expects, assuming practical parameters for the electron cooler, no instability for positively charged particles.

3.3 Longitudinal dispersion relation and ways to avoid instability

The border between a stable situation and instability is reached, if a self-consisting solution, consisting of :

- a (small) perturbation of the beam w.r.t. the equilibrium and
- an additional field acting on the beam

exiting each other, exists. The dispersion relation, describing the border of stability for longitudinal coasting beams are derived in [11, 12].

One way to determine the threshold at which a coasting beam becomes instable in the longitudinal phase space is to consider the Beam Transfer Function (BTF), i.e. the way the beam responds to a small external excitation. The instability threshold is reached if there is an excitation frequency at which the response tends to infinity. First we ignore the additional field seen by the beam due to the longitudinal impedance of the accelerator. This will give an expression valid for very low intensities. A beam with a distribution in relative momentum offset δ given by $f(u = \delta/\Delta\delta)/\Delta\delta$ is excited by a longitudinal field $U/L = (U_0/L) \cdot e^{i(n \cdot s/R - \Omega t)}$. Note that a positive voltage denotes acceleration of a positive charge. $\Delta\delta$ is a typical value for the relative momentum spread (in fact for the stability diagrams in Figs. 2 and 5 shown later in this report, it denotes the RMS width) of the beam. The frequency Ω of the exitation is close to a multiple of the revolution frequency $\Omega \approx n \cdot \omega_0$. In the case of a coasting beam and a very small excitation, the beam response of the of the beam is a single current component at the frequency of the excitation $I = r_{||,0} \cdot U$, where the beam transfer function is given by :

$$r_{||,0} = -\frac{\xi_{||}}{n} \frac{1}{\eta (\Delta\delta)^2} \cdot \hat{f}(\hat{u}) \qquad \text{with} \qquad \hat{u} = \frac{1}{-\eta\, \omega_0 \cdot \Delta\delta} (\frac{\Omega}{n} - \omega_0)$$

where

$$\hat{f}(\hat{u}) = i \int_{P.V.} du \frac{f'(u)}{u - \hat{u}} + sign(\eta) \pi f'(\hat{u}) \qquad \text{and} \qquad \xi_{||} = \frac{N q^2\, \omega_0}{2\pi R \beta\gamma E_0/c} \quad ,$$

and the function *sign* yields the sign of the argument.

The effect of the longitudinal coupling impedance is to create an additional electric field, which in turn contributes to the excitation of the beam. Note that the additional field decelerates for positive charge and voltage. Thus one obtains for the response of the beam $I = r_{||,0}(U_0 - Z_{||} \cdot I)$, and the BTF taking the impedance into account becomes :

$$r_{||,c} = \frac{I}{U_0} = \frac{r_{||,0}}{1 + r_{||,0} \cdot Z_{||}} \quad .$$

The beam is at the instability threshold if the beam transfer function becomes infinite for a particular exciting frequency Ω, i.e. for a value of the parameter $\hat{u}$. In general, the impedance does not change significantly inside the frequency interval considered and thus is assumed constant for the following considerations. The inverse of the function $\hat{f}(\hat{u})$, plotted as a function of the parameter $\hat{u}$, traces lines in a stability diagram, which is shown for 4 distribution functions in Fig. 2. The distribution functions underlying the curves in the stability digram are the solid lines in Fig. 1. If the parameter $U + iV := \xi_{||}(1/(\eta\,\Delta\delta^2))(Z_{||}/n)$ is on the line, the instability threshold is just reached. Points inside the curves are stable.

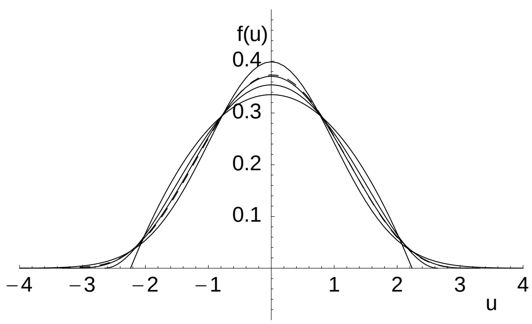

Figure 1: Distributions used to compute stability diagrams. The distribution functions are scaled such that the RMS is unity. Thus one obtains :

$$\begin{aligned} f(u) &= \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \quad , \\ f(u) &= \frac{3}{4\sqrt{5}} \left(1 - \frac{u^2}{5}\right) \quad , \\ f(u) &= \frac{15}{16\sqrt{7}} \left(1 - \frac{u^2}{7}\right)^2 \text{ and} \\ f(u) &= \frac{315}{256\sqrt{11}} \left(1 - \frac{u^2}{11}\right)^4 \end{aligned}$$

for a Gaussian, a parabolic, a parabolic squared and and a parabolic to the forth power. In addition a parabolic to the forth with a small additional tail at the left side is plotted as dashed line, but nearly indistinguishable from the parabolic to the forth power without tail.

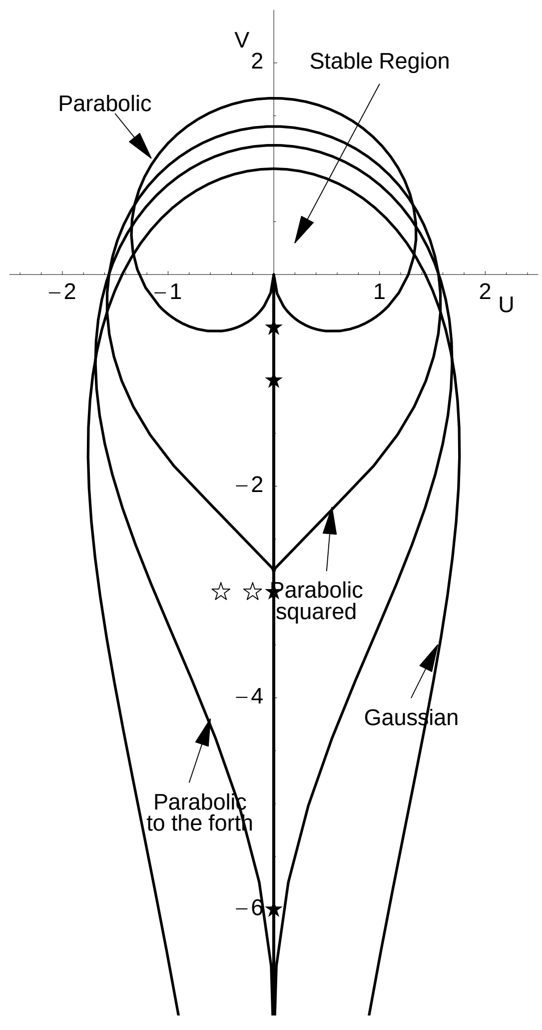

Figure 2: Longitudinal stability diagram for different distribution functions.

From the dispersion relation and the stability diagram, the following conclusions can be drawn :

- Given the large capacitive space-charge impedance (large positive imaginary part) in typical cooling rings, the accelerator should be operated with a negative momentum slip factor η, i.e. below transition. Then the quantity $U + iV$ is located in "thermometer shaft" (which points downwards due to the choice of the time dependence $e^{-i\Omega t}$) of the instability diagram.
- A large $|\eta|$ factor improves stability because, for a given momentum spread, it increases the spread in revolution frequences and thus enhances Landau damping.
- A large momentum spread leads to a more stable situation. At CELSIUS, longitudinal instabilities (self-bunching) have been cured by artificially increasing the momentum spread of the beam [10]. To this end, an voltage alternating between values has been applied to the drift tube of the electron cooler and thus the velocity of the cooling electrons alternates rapidly between two values.

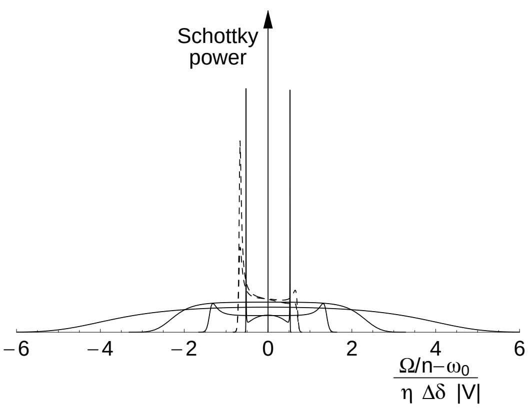

Figure 3: Anomalous Schottky spectra, deformed by the longitudinal coupling impedance of the accelerator. The spectra are parabolic to the forth distribution with an RMS width of $\Delta\delta$ and for values $U + iV$ as indicated in Fig. 4. The solid lines are for purely capacitive impedance with values of $U + iV$: $-i/2$, $-3i$ and $-6i$. The dashed lines show the effect of an additional small real part of the impedance with $U + iV$: $-3i - 0.2$ and $-3i - 0.5$.

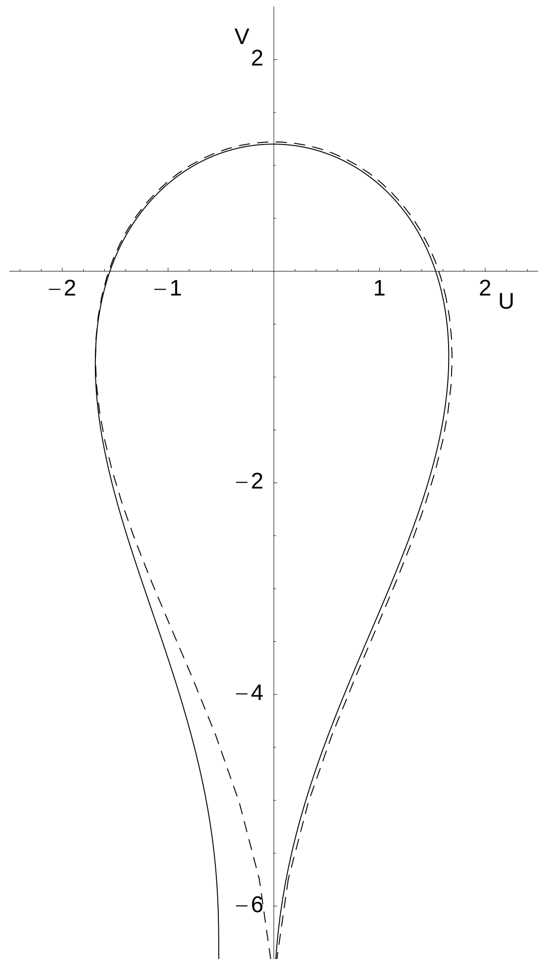

Figure 4: Effect of a small tail at low frequency (i.e. low momenta below transition).

It should be noted that few problems with longitudinal instabilities in cooler rings have been reported, provided they work below transition energy. A possible explanation are stabilizing tails created by "micro"-instabilities at the border of the momentum distribution. Indeed, anomalous Schottky spectra can be observed, where a strong signal occurs at frequencies, which correspond to a small distance between the quantity $U + iV$ and the curve in the stability diagram. If the signal at a given frequency is $I_{s,0}$ without impedance, it is modified due to additional field created via the impedance : $I_s = I_{s,0} - r_{||,0} Z_{||} I_s$.

Taking into account that the signal is proportional to the root of the density and that the power is proportional to the square of the amplitude, one obtains the following behaviour for the Schottky power : $P_s \propto \frac{f(u)}{|1+(U+iV)\hat{f}(\hat{u})|^2}$. This is plotted for the parabolic to the forth distribution in Fig. 3. The large signal observed in the Schottky spectrum denotes also that the beam is excited by strong noise at these frequencies and is thus heated by "micro"-instabilities. The effect of tails on the stability diagrams is shown in Fig. 4. The curve belonging to the parabolic to the forth distribution is shown as a dashed line. If a small tail (containing less than 5% of the particles, see the dashed curve in Fig. 1) is added while keeping the RMS width unchanged, one obtains the solid line with more space to accomodate the beam with a large space-charge and a small positive resistive impedance.

The cooling force itself can also have a stabilising effect. A model is described in reference [13] for the case where the cooling force can be interpreted as a friction force (electron or laser cooling) and the distribution as an equilibrium between cooling and diffusion (e.g. due to intra beam scattering). However, the stabilization due to the cooling force itself is only significant in the case of very low intensities and/or high cooling forces and, thus, negligible for most practical cases.

3.4 Transverse dispersion relation and ways to avoid instability

To investigate the border of transverse stability, one can proceed in a manner analogeous to the longintudinal case and look for a self-consisting solution consisting of an exciting field and a perturbation of the beam with respect to equilibrium. Derivations for the case without taking the electron cooling friction force into account are given in references [12, 14].

The beam responds significantly to an external excitation only if the frequency is close to one of the transverse sidebands :

fast waves	:	$\Omega \approx (n+Q)\omega_0$ with $n > -Q$
slow waves	:	$\Omega \approx (n-Q)\omega_0$ with $n > Q$

The definitions are such that the frequency Ω is always positive, but for the "fast" waves there are a few modes with negative n describing waves propagating in the Backward direction. A friction force (electron cooling) can be taken into account in the single-particle equations of motion as an additional damping term. This can be justified, if Landau damping comes from a momentum spread.

First, the Beam Transfer Function in the low-intensity limit is given. Then the beam is excited by an externally applied voltage creating a transverse electric field, and the single-particle equation of motion becomes :

$$\frac{d^2}{dt^2}x(s,t) + Q^2\omega_r^2 x(s,t) + \zeta\frac{d}{dt}x(s,t) = \frac{qU(s,t)}{\gamma E_0/c^2} = \frac{q\hat{U}}{\gamma E_0/c^2}e^{i(n\cdot s/R-\Omega t)}$$

where ζ is a parameter describing the damping of single-particle oscillations due to the electron cooler, $Q = (1+\xi\delta)$ the tune and $\omega_r = (1-\eta\delta)\omega_0$ the revolution (angular) frequency. The transverse deflecting field has to be evaluated at the instantaneous longitudinal particle position, which obeys $ds/dt = R\cdot\omega_r$. The parameter ζ can be approximated by :

$$\zeta = -\frac{l_c}{L}\frac{1}{\gamma E_0/c^2}\frac{dF'_\perp}{dv'_\perp} = \frac{l_c/L}{\gamma E_0/c^2}\frac{4\pi q^2 e^2}{m_e(4\pi\epsilon_0)^2}n_e L_c\frac{\sqrt{\pi}}{8}\frac{1}{(k\cdot T_\perp/m_e)^3/2}$$

neglecting the variations of the betatron function around the accelerator, and any enhancement of the cooling rate due to a finite dispersion function. Here $dF'_\perp/dv'_\perp$ is the derivative of the transverse friction force w. r. t. the transverse velocity in a coordinate system moving with the beam. For the last transformation, the expression for the transverse cooling force for small velocities, as given e.g. in references [15, 16], is inserted.

The resulting mean displacement times current $I\bar{x} = (i/\beta)\, r_{\perp,0}\, U$, for a beam with a distribution $f(\delta/\Delta\delta)/\Delta\delta$ in relative momentum deviation is expressed by :

$$r_{\perp,0} = -\xi_\perp\frac{1}{|\xi Q_0 - \eta(Q_0 \pm n)|\Delta\delta}\hat{f}_\perp(\hat{u},\hat{\zeta}) \qquad \text{with} \qquad \xi_\perp = \frac{Nq^2\beta}{4\pi Q_0\omega_0 L\gamma E_0/c^2} \qquad ,$$

$$\hat{u} = \mp \frac{(n \pm Q_0) - \Omega/\omega_0}{(\xi Q_0 \mp (n \pm Q_0)\eta)\Delta\delta} \quad , \quad \hat{\zeta} = \pm \frac{\zeta/2}{(\xi Q_0 \mp (n \pm Q_0)\eta)\Delta\delta} \quad ,$$

and :

$$\hat{f_\perp}(\hat{u}, \hat{\zeta}) = sign(\xi Q_0 \mp (n \pm Q_0)\eta) \cdot i \int du \frac{f(u)}{u - (\hat{u} + i\hat{\zeta})} \quad .$$

The upper (lower) sign applies for the fast (slow) waves. The above expression is valid for positive ζ.

The analogeous expression for accelerators without cooling can be computed taking the limit $\zeta \to 0^+$ ($\zeta \to 0^-$ would lead to a result violating causality), leading to the more familiar expression :

$$\hat{f_\perp}(\hat{u}, \hat{\zeta} = 0) = sign(\xi Q_0 \mp (n \pm Q_0)\eta) \cdot i \int_{P.V.} du \frac{f(u)}{u - \hat{u}} \quad \mp \quad \pi f(\hat{u}) \quad .$$

Again, as in the longitudinal case, the effect of the coupling impedance is to create an additional field, which can be described by an additional voltage $-(\beta/i)\, Z_\perp(\Omega)\, I\bar{x}$. In addition, the effect of a transverse damper is taken into account via an additional field $\pm(2\pi i\, \beta\gamma E_0)/(Nq^2 c)\; R\, D\, I\bar{x}$ in smooth approximation, ignoring the modulation of the betatron function around the machine. The constant D is a damper gain factor denoting the kick applied at the kicker divided by the detected mean position at the pick-up. Taking all voltages into account, one ends up with the relation :

$$I\bar{x} = \frac{i}{\beta}\, r_{\perp,0}(U - \frac{\beta}{i} Z_\perp(\Omega)\, I\bar{x} \pm \frac{2\pi i\, \beta\gamma E_0}{Nq^2 c} R \cdot D \cdot I\bar{x}))$$

The beam transfer function, taking the impedance into account, becomes :

$$r_{\perp,c} = \frac{\beta}{i} \frac{I\bar{x}}{U_0} = \frac{r_{\perp,0}}{1 + r_{\perp,0}(Z_\perp \pm r_{\perp,0} \frac{2\pi\, \gamma E_0}{Nq^2 c} RD)}$$

The beam is at the instability threshold for a given mode if the beam transfer function $r_{\perp,c}$ becomes infinity. This leads to the dispersion relation :

$$U + iV := \xi_\perp \frac{Z_\perp}{|\xi Q_0 - \eta(Q_0 \pm n)|\Delta\delta} = \frac{1}{\hat{f_\perp}(\hat{u}, \hat{\zeta})} \mp \hat{d} \quad \text{with} \quad \hat{d} = \frac{\beta c}{4\pi\omega_0 Q_0} \frac{D}{|\xi Q_0 \mp \eta(n \pm Q_0)|\Delta\delta}$$

The border between stability and instability is reached, when there is a particular value of the frequency Ω, for which the the dispersion relation is satisfied.

The complex function $1/\hat{f_\perp}(\hat{u}, \hat{\zeta}) \mp \hat{d}$ plotted for any $\hat{u}$ traces curves in the $U + iV$ plane, which are shown in the stability digram given in Fig. 5. The curves belonging to fast waves and no cooling (i.e. $\hat{\zeta} = 0$ are drawn as dashed lines. The limit for the slow waves without cooling correspond to the solid lines. In addition, the curves for slow and fast waves with an alectron cooling force $|\hat{\zeta}| = 0.1$ are plotted as dot-dashed lines. Stability corresponds to the region on the left (right) side of the curves for the slow (fast) waves.

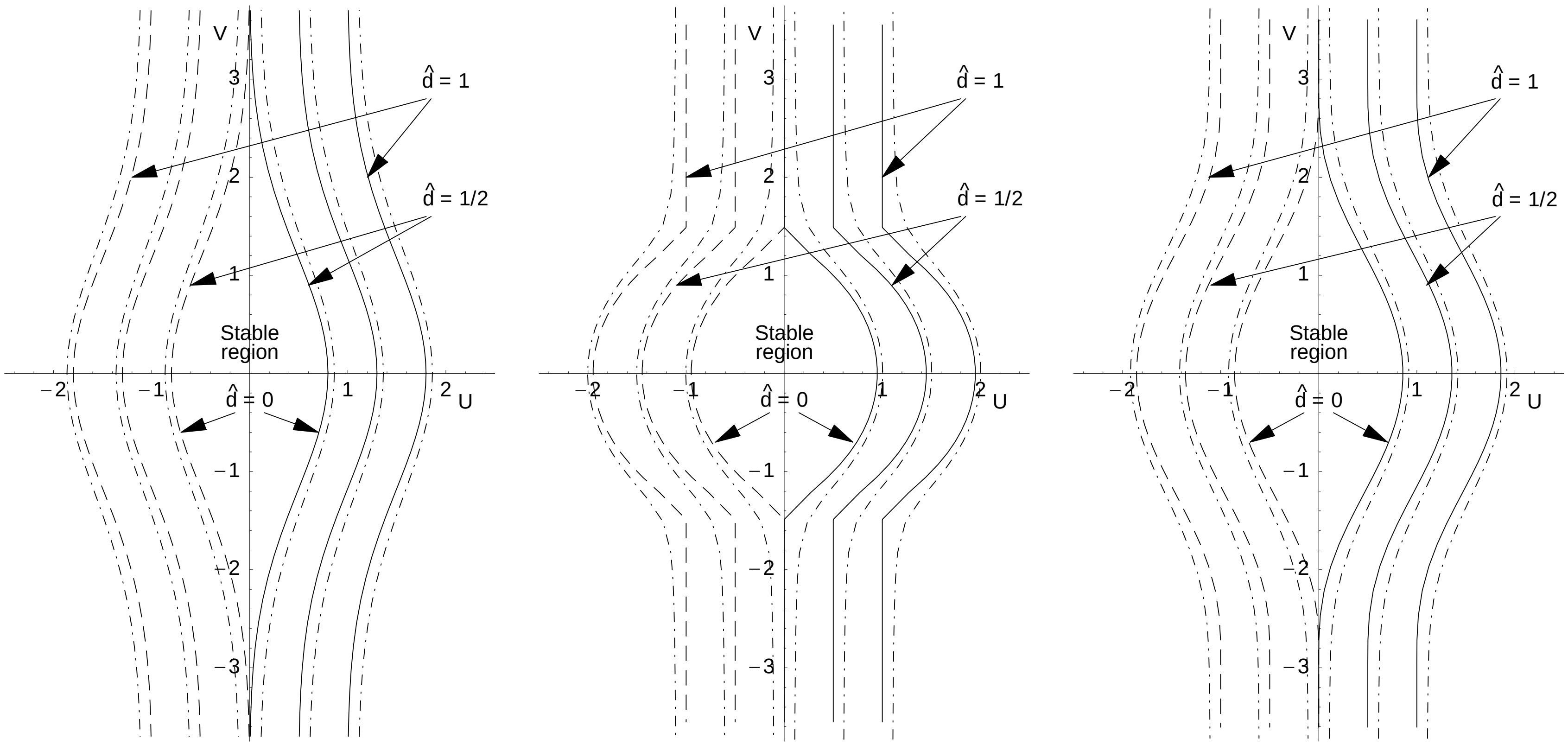

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Figure 5: Transverse stability diagrams taking the damping due to an electron cooling system and action of a damper into account. Solid (dashed) lines are for slow (fast) waves without cooling force. Dot-dashed lines take into addition a friction force into account with the parameter $\hat{\zeta} = 0.1$. The three images are for Gaussians (a), parabolic distributions (b) and parabolic to the forth power (c) of the function $f(u)$.

From the dispersion relation and the stability diagrams in Fig. 5, one can draw the following conclusions :

- In case of a positive real part of the impedance, i.e. in most practical cases, fast waves do not lead to instability. An exception may be, in the case of particles with a negative charge (e.g. antiprotons) impedances due to electrons captured in the beam potential.
- Slow waves can become unstable in the case of a positive real part of the impedance. Provided that the real part of the impedance is small, a capacitive space-charge impedance larger than expected from simple estimations can be tolerated.
- One option to stabilize the beam is to increase the factor $|\xi Q_0 - \eta(Q_0 \pm n)|\Delta\delta$. Note that this factor depends on n and should be sufficiently large for any $n > Q$. In case of the slow waves, the first factor becomes large with a chromaticity which has the same sign than the momentum compaction factor η. This means that a "typical" cooler ring working below transition should be operated with negative chromaticities ξ.

  Increase of the relative momentum spread $\Delta\delta$ stabilizes the beam. This can be achieved by longitudinal heating either by filtered noise applied to the beam, or by changing periodically (fast enough that the beam cannot follow as a whole the instananeous velocity of the electrons) the velocity of the cooling electrons.
- A friction force (e.g. due to an electron cooler) has a stabilizing effect. However, in most practical cases, the effect is small. It might become significant for beams with very low intensities and very small emittances.
- A transverse damper is a suitable device to damp coherent transverse oscillations. It should be mentioned that a stochastic cooling system acts like a damper with a particularly high gain (In fact, the gain is even so high that the smooth approximation to include the damper in the computation of the beam transfer function becomes questionable). Thus, in case of a ring with stochastic cooling, instabilities are effectively damped, if the mode frequency is inside its frequency band.

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A Symbols and Notation

Ω	frequency of excitation or of potentially unstable mode (Note that the time dependence $e^{-i\Omega t}$ is adopted.)
β, γ	relativistic β and γ factors
N	number of beam particles
q, E_0	electric charge and rest energy of pericles (for protons $1.601 \cdot 10^{-19}$ As and 938 MeV)
a, b	radius of the beam and the (circular) vacuum chamber
R, L	mean radius and circumference of the accelerator $L = 2\pi R$
ϵ	RMS emittance of the beam - surface in phase space divided by π, i.e. $\epsilon = \sigma^2/\beta_c$ with σ the RMS beam width and β_c the (Twiss) betatron function,
F	form factor $F = 1/\sqrt{2\pi}$ for a Gaussian beam with ϵ the RMS emittance
$Z_{\|\|}$, $Z_\perp$	longitudinal and transverse coupling impedances
Z_0	impedance of free space $Z_0 \approx 377\Omega$
Q_{cav}, ω_{cav}	Quality factor and resonant (angular) frequency of a cavity (for broad-band impedance $Q_{cav} \approx 1$ and ω_{cav} near the cut-off of the vacuum chamber).
δ, $\Delta\delta$	relative momentum deviation of a particle and relative (RMS) momentum spread of the beam
Q, ω_0	transverse tune and revalution (angular) frequency of on momentum particle
η	momentum slip factor (negative below transition)
ξ	chromaticity $\xi := (1/Q)(dQ/d\delta)$
c	velocity of light
l_c	length of an electron cooler
n_e	electron density inside a cooling electron beam in a co-moving coordinate system
e, m_e	electron charge and mass
ϵ_0	dielectric constant in free space
$k, T_\perp$	Boltzman constant and transverse temperature of an electron beam
L_C	Coulomb logaritm for the computation of the electron cooling friction force