Comparison of transfer maps of PTC and MADX for the dipole magnets: SBENDS and RBENDS

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Abstract:

Review of the two types of bending magnets: SBEND and RBEND to get an understanding of the transfer maps under different conditions.

Table of Contents

Definition of transfer maps in PTC and MADX

The first and second order transfer MAPs for PTC and MADX are defined in the following way:

$$\begin{pmatrix} x_1 \\ px_1 \\ y_1 \\ py_1 \\ t_1 \\ pt_1 \end{pmatrix} = \begin{pmatrix} R11 & R12 & R13 & R14 & R15 & R16 \\ R21 & R22 & R23 & R24 & R25 & R26 \\ R31 & R32 & R33 & R34 & R35 & R36 \\ R41 & R42 & R43 & R44 & R45 & R46 \\ R51 & R52 & R53 & R54 & R55 & R56 \\ R61 & R62 & R63 & R64 & R65 & R66 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ px_0 \\ y_0 \\ py_0 \\ t_0 \\ pt_0 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ px_2 \\ y_2 \\ py_2 \\ t_2 \\ pt_2 \end{pmatrix} = \begin{pmatrix} \mathbf{T11} & \mathbf{T12} & \mathbf{T13} & \mathbf{T14} & \mathbf{T15} & \mathbf{T16} \\ \mathbf{T21} & \mathbf{T22} & \mathbf{T23} & \mathbf{T24} & \mathbf{T25} & \mathbf{T26} \\ \mathbf{T31} & \mathbf{T32} & \mathbf{T33} & \mathbf{T34} & \mathbf{T35} & \mathbf{T36} \\ \mathbf{T41} & \mathbf{T42} & \mathbf{T43} & \mathbf{T44} & \mathbf{T45} & \mathbf{T46} \\ \mathbf{T51} & \mathbf{T52} & \mathbf{T53} & \mathbf{T54} & \mathbf{T55} & \mathbf{T56} \\ \mathbf{T61} & \mathbf{T62} & \mathbf{T63} & \mathbf{T64} & \mathbf{T65} & \mathbf{T66} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_0 \\ \mathbf{px}_0 \\ \mathbf{y}_0 \\ \mathbf{py}_0 \\ \mathbf{t}_0 \\ \mathbf{pt}_0 \end{pmatrix}$$

$$\mathbf{T11} = (T111, T112, T113, T114, T115, T116)$$
$$\mathbf{T12} = (T121, T122, T123, T124, T125, T126)$$

... etc...

and $\mathbf{x}_0 = \begin{pmatrix} x_0 \cdot x_0 \\ x_0 \cdot px_0 \\ x_0 \cdot y_0 \\ x_0 \cdot py_0 \\ x_0 \cdot t_0 \\ x_0 \cdot pt_0 \end{pmatrix}$, $\mathbf{px}_0 = px_0 \begin{pmatrix} x_0 \\ px_0 \\ y_0 \\ py_0 \\ t_0 \\ pt_0 \end{pmatrix}$, , , $\mathbf{pt}_0 = pt_0 \begin{pmatrix} x_0 \\ px_0 \\ y_0 \\ py_0 \\ t_0 \\ pt_0 \end{pmatrix}$

As an example, X2 equals:

$$\begin{aligned} x2 = {} & T111 \cdot x_0 \cdot x_0 + T112 \cdot x_0 \cdot px_0 + T113 \cdot x_0 \cdot y_0 + T114 \cdot x_0 \cdot py_0 + T115 \cdot x_0 \cdot t_0 + T116 \cdot x_0 \cdot pt_0 \\ & + T121 \cdot px_0 \cdot x_0 + T122 \cdot px_0 \cdot px_0 + T123 \cdot px_0 \cdot y_0 + T124 \cdot px_0 \cdot py_0 + T125 \cdot px_0 \cdot t_0 + T126 \cdot px_0 \cdot pt_0 \\ & + T131 \cdot y_0 \cdot x_0 + T132 \cdot y_0 \cdot px_0 + T133 \cdot y_0 \cdot y_0 + T134 \cdot y_0 \cdot py_0 + T135 \cdot y_0 \cdot t_0 + T136 \cdot y_0 \cdot pt_0 \\ & + T141 \cdot py_0 \cdot x_0 + T142 \cdot py_0 \cdot px_0 + T143 \cdot py_0 \cdot y_0 + T144 \cdot py_0 \cdot py_0 + T145 \cdot py_0 \cdot t_0 + T146 \cdot py_0 \cdot pt_0 \\ & + T151 \cdot t_0 \cdot x_0 + T152 \cdot t_0 \cdot px_0 + T153 \cdot t_0 \cdot y_0 + T154 \cdot t_0 \cdot py_0 + T155 \cdot t_0 \cdot t_0 + T156 \cdot t_0 \cdot pt_0 \\ & + T161 \cdot pt_0 \cdot x_0 + T162 \cdot pt_0 \cdot px_0 + T163 \cdot pt_0 \cdot y_0 + T164 \cdot pt_0 \cdot py_0 + T165 \cdot pt_0 \cdot t_0 + T166 \cdot pt_0 \cdot pt_0 \end{aligned}$$

where:

px	py
$px = (1+\delta) \cdot x' / \sqrt{1 + x'^2 + y'^2}$	$py = (1+\delta) \cdot y' / \sqrt{1 + x'^2 + y'^2}$

Table 1. Definition of px and py used by both PTC and MADX. MADX only use these definitions of px and py for the transfer MAPS; for optics calculations MADX use $px = x'$ and $py = y'$.

(x, x', y, y', l and $\delta = \frac{\delta p}{p}$ are defined as in TRANSPORT, e.g. x'=dx/ds; see reference [3]) and

t	pt
$t^{**} = \frac{l}{\beta} + \frac{L_0 + l}{\beta} \cdot \frac{\left[\beta \cdot \sqrt{(1+\delta)^2 + \frac{(1-\beta^2)}{\beta^2}} - (1+\delta)\right]}{(1+\delta)}$	$\mathrm{pt} = \frac{\Delta E}{p \cdot c}$ ***

Table 2. Definition of t and pt in PTC and MAX. $\beta = \frac{\mathrm{v}}{\mathrm{c}}$ and L_0 is the design length. Notice that MADX only use the above definitions for the transfer MAPS; for optics calculations, MADX use $t^{****} = \frac{(L_0 - l)}{\beta}$ and $\mathrm{pt} = \delta \cdot \beta$.

** PTC can calculate the variables "t" and "pt" differently, depending on the settings. In this report is always used:* ptc_create_layout,model=2,method=6,nst=5,time=true,exact; *which corresponds to a 5D case and* $pt = \Delta E / p \cdot c$

*** The formula for "t" is copied from reference [4]. The formula for "t" could not be tested when calling PTC from MADX. The reason is that only in 6D cases will "t" be calculated, but the integration of PTC in MADX only allows 6D cases when cavities are included – and our model consists of only a bending magnet. As a result, t is always zero for PTC.*

**** For high energy beams,* $\mathrm{pt} = \frac{\Delta E}{c \cdot p} \cong \delta \cdot \beta$ *(See reference [8])*

***** The documentation gives the following definition:* "T: Velocity of light times the negative time difference with respect to the reference particle: T = - *c t*, [m]. A positive T means that the particle arrives ahead of the reference particle." *(See reference [9])*

The first and second order calculations can be combined to give the result after the transfer through the magnet. Again this is done differently in PTC and MADX:

PTC	$\begin{pmatrix} x \\ px \\ y \\ py \\ t \\ pt \end{pmatrix} = \begin{pmatrix} x_1 \\ px_1 \\ y_1 \\ py_1 \\ t_1 \\ pt_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ px_2 \\ y_2 \\ py_2 \\ t_2 \\ pt_2 \end{pmatrix} + \begin{pmatrix} x_3 \\ px_3 \\ y_3 \\ py_3 \\ t_3 \\ pt_3 \end{pmatrix}^{*} + .. + ..$
	Matrices calculated for x=0, px=0, , , δ =0
MADX	$\begin{pmatrix} x \\ px \\ y \\ py \\ t \\ pt \end{pmatrix} = \begin{pmatrix} x_1 \\ px_1 \\ y_1 \\ py_1 \\ t_1 \\ pt_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ px_2 \\ y_2 \\ py_2 \\ t_2 \\ pt_2 \end{pmatrix}^{**}$

Table 3. Combination of transfer maps, to give the final result at the exit of the element. The combinations are given for both PTC and MADX. The transfer maps from MADX are only calculated up to second order.

**   PTC is a real integrator; it does not use matrices for calculation. The transfer maps are derived from the final result. Note also that the ptc_twiss command by default give the results at the end of the elements and not at the start nor in the middle.*

*** The first and second order transfer maps from MADX - when they are calculated On-Axis(see page 5) - are identical to the first and second order transfer maps from PTC. Therefore, when we do matrix calculations, we need to use the same definitions of the input parameters (i.e. x,px,y,py,t and pt) as PTC.*

Comparison of first order maps from PTC and MADX

The transfer maps are generated with the following commands:

PTC	`ptc_normal,no=3,icase=5,maptable;`
MADX	`twiss , sectormap, RMATRIX, BETA0=INITBETA0 , DELTAP = PSHIFT , file=optics.out , sectorfile=secmap;`

Table 4. MADX command used to create transfer maps. Shown both for PTC when it is called from MADX and from MADX itself.

The transfer maps coming directly from PTC and MADX are difficult to read. They can be converted to TFS format (See reference [6]).

The transfer maps from PTC and MADX are identical, when the input parameters are zero i.e. when x, px, y, py and DELTAP ($\delta = \frac{\delta p}{p}$) all equals zero.

This will in the following be referred to as “On-Axis” (*The opposite will be referred to as “Off-Axis”*)

In MADX, the first order transfer maps changes when any of the input parameters (x,px,y,py or δ) changes. Contrary to MADX, in PTC, the first order maps always remain constant, independent of input parameters.

Example of MADX, where the first order maps change, when the input variables change.

relativistic beta = 0.99999987 and δ=0:

First order map for a dipole: SBEND, L=1, ANGLE=0.04 with (x0=0, px0=0)

"R11R61"	0.99920	-0.00160	0.00000	0.00000	-0.03999	0.00000
"R12R62"	0.99973	0.99920	0.00000	0.00000	-0.02000	0.00000
"R13R63"	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
"R14R64"	0.00000	0.00000	1.00000	1.00000	0.00000	0.00000
"R15R65"	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
"R16R66"	0.02000	0.03999	0.00000	0.00000	-0.00027	1.00000

Table 5. This is the on-axis first order map from MADX. It is identical to the first order map from PTC

First order map for a dipole: SBEND, L=1, ANGLE=0.04 with (x0=0.1, px0=0.03)

"R11R61"	1.00039	-0.00160	0.00000	0.00000	-0.03999	0.00000
"R12R62"	1.00433	0.99800	0.00000	0.00000	-0.04997	0.00000
"R13R63"	0.00000	0.00000	0.99880	-0.00001	0.00000	0.00000
"R14R64"	0.00000	0.00000	1.00460	1.00119	0.00000	0.00000
"R15R65"	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
"R16R66"	-0.00983	0.03999	0.00000	0.00000	-0.00027	1.00000

Table 6. The first order map from MADX changes with the input parameters

Even though the first order transfer map changes with the input parameters, the determinant of the first order map always stays 1 independent of the input parameters i.e. Determinant[“FIRST ORDER MAP”] ≡1. This is a result of Liouville’s theorem.

Example of MADX, where the first order maps change, when $\delta = \frac{\Delta p}{p}$ change.

relativistic $\beta = 0.8$ and $\delta = \frac{\Delta p}{p} = 0.01$

dipole : SBEND, L=1, ANGLE=0.04;

PTC:

"R11R61"	0.99920	-0.00160	0.00000	0.00000	-0.04999	0.00000
"R12R62"	0.99973	0.99920	0.00000	0.00000	-0.02500	0.00000
"R13R63"	0.00000	0.00000	1.00000	-0.00001	0.00000	0.00000
"R14R64"	0.00000	0.00000	1.00000	1.00119	0.00000	0.00000
"R15R65"	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
"R16R66"	0.02500	0.04999	0.00000	0.00000	0.56208	1.00000

Table 7. The first order transfer maps from PTC. Equal to the matrix from MADX with δ=0

MADX:

"R11R61"	0.99922	-0.00158	0.00000	0.00000	-0.04981	0.00000
"R12R62"	0.99974	0.99920	0.00000	0.00000	-0.02515	0.00000
"R13R63"	0.00000	0.00000	1.00000	0.00002	0.00000	0.00000
"R14R64"	0.00000	0.00000	1.00000	1.00002	0.00000	0.00000
"R15R65"	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
"R16R66"	0.02466	0.04981	0.00000	0.00000	0.55100	1.00000

Table 8. The first order transfer from MADX with δ=0.01

Notice the following differences between the tables:

- The MADX table is corrected according to the different definition of "px", e.g. The matrix element R21= $-0.00158 \cdot (1+0.01) = -0.00160$ (rounded), when $\delta = 0.01$
- The matrix elements: R16, R26, R51, and R52 in table 7 and in table 5 (on page 5) are exactly inversely proportional to the relativistic beta (here $\beta = 0.8$). The matrices from PTC change when β change, but stays constant when δ changes.

The first order maps from PTC and MADX for on-axis and with relativistic beta~1, are identical to the first order map from TRANSPORT

First order map for a dipole: SBEND, L=1, ANGLE=0.04 with (x0=0, px0=0)

PTC/MADX: relativistic beta ~ 1

```
"R11R61"  0.99920011 -0.00159957  0.00000000  0.00000000 -0.03998934  0.00000000
"R12R62"  0.99973335  0.99920011  0.00000000  0.00000000 -0.01999734  0.00000000
"R13R63"  0.00000000  0.00000000  1.00000000  0.00000000  0.00000000  0.00000000
"R14R64"  0.00000000  0.00000000  1.00000000  1.00000000  0.00000000  0.00000000
"R15R65"  0.00000000  0.00000000  0.00000000  0.00000000  1.00000000  0.00000000
"R16R66"  0.01999734  0.03998934  0.00000000  0.00000000 -0.00026639  1.00000000
```

transposed:

"R11R61"	"R12R62"	"R13R63"	"R14R64"	"R15R65"	"R16R66"
0.99920000	0.9997330	0.00000000	0.00000000	0.00000000	0.01999730
-0.00159957	0.9992000	0.00000000	0.00000000	0.00000000	0.03998930
0.00000000	0.0000000	1.00000000	1.00000000	0.00000000	0.00000000
0.00000000	0.0000000	0.00000000	1.00000000	0.00000000	0.00000000
-0.03998930	-0.0199973	0.00000000	0.00000000	1.00000000	-0.00026639
0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	1.00000000

The program **TRANSPORT** [1] (See reference [3]), gives the following first order matrix:

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \\ l_1 \\ \delta_1 \end{pmatrix} = \begin{pmatrix} 0.99920 & 0.99973 & 0.00000 & 0.00000 & 0.00000 & 0.02000 \\ -0.00160 & 0.99920 & 0.00000 & 0.00000 & 0.00000 & 0.03999 \\ 0.00000 & 0.00000 & 1.00000 & 1.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00000 & 0.00000 \\ -0.03999 & -0.02000 & 0.00000 & 0.00000 & 1.00000 & -0.00027 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ l_0 \\ \delta_0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \\ l_1 \\ \delta_1 \end{pmatrix} = \begin{pmatrix} \cos k_x L & \frac{1}{k_x}\sin k_x L & 0 & 0 & 0 & \frac{h}{k_x^2}[1-\cos k_x L] \\ -k_x \sin k_x L & \cos k_x L & 0 & 0 & 0 & \frac{h}{k_x}\sin k_x L \\ 0 & 0 & \cos k_y L & \frac{1}{k_y}\sin k_y L & 0 & 0 \\ 0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\ -\frac{h}{k_x}\sin k_x L & -\frac{h}{k_x^2}[1-\cos k_x L] & 0 & 0 & 1 & -\frac{h^2}{k_x^3}[k_x L - \sin k_x L] \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ l_0 \\ \delta_0 \end{pmatrix}$$

Where:

$$k_x^2 = (1-n)\cdot h^2,\ \ k_y^2 = n\cdot h^2,\ L = \text{Arclength},\ n = 0\ (\text{for dipoles}),\ h = \frac{1}{\rho_0},\ h\cdot L = \text{bending angle}$$

[1]*Notice that TRANSPORT calculates as if relativistic beta ~ 1*

Comparison of second order maps from PTC and MADX

In PTC the second order maps in PTC are always constant - independent of input parameters. MADX on the contrary, change the second order maps when δ changes.

The PTC maps are always calculated for on-axis.

There are two special cases where the second order maps from PTC and MADX are not equal, when they should have been:

1. When the relativistic beta goes below 0.4857. In this case the T566 coefficient in the second order map from MADX jumps to zero. This effect is independent of particle type.

2. When the dipole is a RBEND with a quadrupolar component.

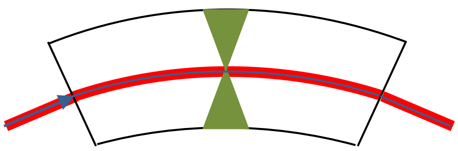
1. The first case is an anomaly in MADX, where the variable T566 jumps down to zero when the relativistic beta is less or equal to 0.4857. It cannot be seen if MADX inherited this "feature" from TRANSPORT, because TRANSPORT always calculates the transfer maps as if relativistic beta ~ 1.

2. The second case where the transfer maps are different in PTC and MADX is for RBEND's with multipolar components. Here is a case of an RBEND with a quadrupolar component K1:

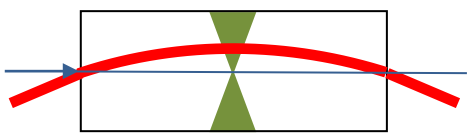
```
dipole  : RBEND,     L=1, ANGLE=0.04, K1=0.1;

PTC:         T111 = -0.00191797  ,     T211 = -0.00009481
MADX:        T111 = -0.00191889  ,     T211 = -0.00009654
TRANSPORT:   T111 = -1.919E-03   ,     T211 = -9.654E-05
```

(The complete transfer maps for second order, for all the three programs: PTC, MADX and TRANSPORT, are in appendix A.)

The second order map from MADX corresponds exactly to the second order map from TRANSPORT (See reference [3]). This is to be expected as MADX is derived from TRANSPORT. This second case with RBEND+K1, where the second order transfer map is different for PTC and MADX, is known. PTC does not implement wedges with multipole components in the same way as MADX, as the wedges are too ill-defined in that case (See reference [10]).



PTC: SBEND+K1. The quadrupole is centered around the nominal orbit (blue). The actual orbit is red – it is **not** affected by the quadrupole.



PTC: RBEND+K1. The quadrupole is centered around the nominal orbit (blue). The actual orbit is red – it is **indeed** affected by the quadrupole.

For on-momentum beams, the results from PTC and MADX are equal to second order

When we have on-momentum beams, PTC and MADX give identical results within 2'nd order.

Example, a sector dipole with an angle of 0.04 rad and input parameters (x=0.1, px=0.03). The relativistic beta ~1 - only T depends on the relativistic beta.

```
dipole  : SBEND,     L=1, ANGLE=0.04;
```

	PTC			MADX
X0	0.10000		X0	0.10000
PX0	0.03000		PX0	0.03000
T0	0.00000		T0	0.00000
PT0	0.00000		PT0	0.00000
X	0.13005		X	0.13004
PX	0.02980		PX	0.02980
T	0.00000		T	-0.00505
PT	0.00000		PT	0.00000

Table 9. PTC and MADX are equal to second order.

The small difference of X between PTC and MADX, is explained by the third order coefficient x3:

The non-zero third order coefficients are (from the second order maps from PTC) :

```
    13   -.95753954298421E-25     3      3 0  0 0   0
    14   0.35839567034718E-21     3      2 1  0 0   0
    15   -.79957342434515E-03     3      1 2  0 0   0
    16   0.49946683730733         3      0 3  0 0   0
```

The overwhelming dominant coefficient is:

```
    16   0.49946683730733         3      0 3  0 0   0
```

X1+x2 =	0.13004056
X3 =	0.0000134856 ~ (0.49946667732866*0.03^3)
X1+x2+x3 =	0.130054
X =	0.13005

Table 10. Adding the 3'rd order component of the position, to the first - and second order, gives the same result as PTC.

That PTC and MADX always give identical results for on-momentum beams have been tested in: DRIFT SPACE, QUADRUPOLE and SBEND+K1. *Equal results, in this context, means that PTC and MADX are equal to within second order.*

Another example of on-momentum beam with combinations of x and px input:

The second order matrix results from MADX and PTC are identical and also identical to the result of MADX. The difference between the result from MADX and the result from PTC are 3'rd order effects. The relativistic beta ~1 - only T depends on the relativistic beta.

dipole : SBEND, L=1, ANGLE=0.4;

	PTC		MADX
X0	0.10000000		0.10000000
PX0	0.30000000		0.30000000
T0	0.00000000		0.00000000
PT0	0.00000000		0.00000000
X1	0.39984002		0.39984002
PX1	0.29960008		0.29960008
T1	-0.00999813		-0.00999813
PT1	0.00000000		0.00000000
X2	0.00209756		0.00209756
PX2	-0.00179952		-0.00179952
T2	-0.04498801		-0.04498801
PT2	0.00000000		0.00000000
X1+X2	0.40193758		0.40193758
PX1+PX2	0.29780056		0.29780056
T1+T2	-0.05498613		-0.05498613
PT1+PT2	0.00000000		0.00000000
X	0.41647246		0.40193758
PX	0.29775813		0.29780055
T	0.00000000		-0.05498613
PT	0.00000000		0.00000000

Table 11. The results from matrix calculations of PTC and MADX are identical. They are also identical to the result from MADX.

Notice the following important observations:

- The results from PTC are different from the results from MADX.
- The results from MADX are identical to the second order calculation.

For off-momentum beams, with px zero, the results from PTC and MADX are equal

In the following example, the second order calculation is done with the definition of pt as it is done in MADX. This will be the only time that the MADX definitions are used in the matrix calculations, but it is done here to make an example.

Example with: $\delta = \frac{\Delta p}{p} = 0.01$ and relativistic $\beta = 0.8$:

```
dipole  : SBEND,    L=1, ANGLE=0.04;
```

	PTC		MADX
X0	0.00000000		0.00000000
PX0	0.00000000		0.00000000
T0	0.00000000		0.00000000
PT0	0.00801431		0.00800000
X1	0.00020033		0.00019997
PX1	0.00040061		0.00039989
T1	0.00450471		0.00449667
PT1	0.00801431		0.00800000
X2	-0.00000237		-0.00000236
PX2	-0.00000072		-0.00000072
T2	-0.00006772		-0.00006748
PT2	0.00000000		0.00000000
X1+X2	0.00019796		0.00019761
PX1+PX2	0.00039989		0.00039917
T1+T2	0.00443699		0.00442918
PT1+PT2	0.00801431		0.00800000
X	0.00019799		0.00019799
PX	0.00039989		0.00039593
T	0.00000000		-0.00451602
PT	0.00801431		0.00000000

* The MADX column is done with a MADX definition of the PT variable. This is done as an example

Table 12. PTC and MADX are equal for off-momentum beams - notice that for MADX, "px" must be multiplied with the factor $(1+\delta)$ - in this case `0.00039593` * (1+0.01)= `0.00039989`. The matrix calculations from MADX are different from the matrix calculations from PTC.

Notice the following important observations:

- When using the MADX definitions for the input variables, and not the correct PTC definitions, then the matrix calculations in MADX give wrong results.
- PT is zero for MADX, but PTC has the correct value for PT.
- The px variable from MADX needs to be multiplied with $(1+\delta)$ to give the same as PTC. The X variables are equal.

Verification of the calculation of "px" in the previous example:

That PTC is correct for off-momentum particles, was checked with Mathematica (see reference [7]), in the following way:

The radius of the design particle is: $R_0 = \frac{L_0}{angle} = \frac{1m}{0.04rad} = 25m$

The radius for the off-momentum particle is: $R = \frac{R_0 \cdot p}{p_0} = 25.25m$ (R is proportional to p)

The trajectory of the design particle follows the equations:
`z0[phi0_]:=R0*Cos[phi0]` where x and z are survey coordinates.
`x0[phi0_]:=R0*Sin[phi0]`

The trajectory of the off-momentum particle follows the equations:
```
z[phi_]:=R*Cos[phi]
x[phi_]:=R*Sin[phi]-(R-R0)
```

The following drawing (not to scale) illustrates the trajectories of the design particle(blue) and the off-momentum particle(rose). Both the design particle and the off-momentum particle start from the right and moves towards the left:

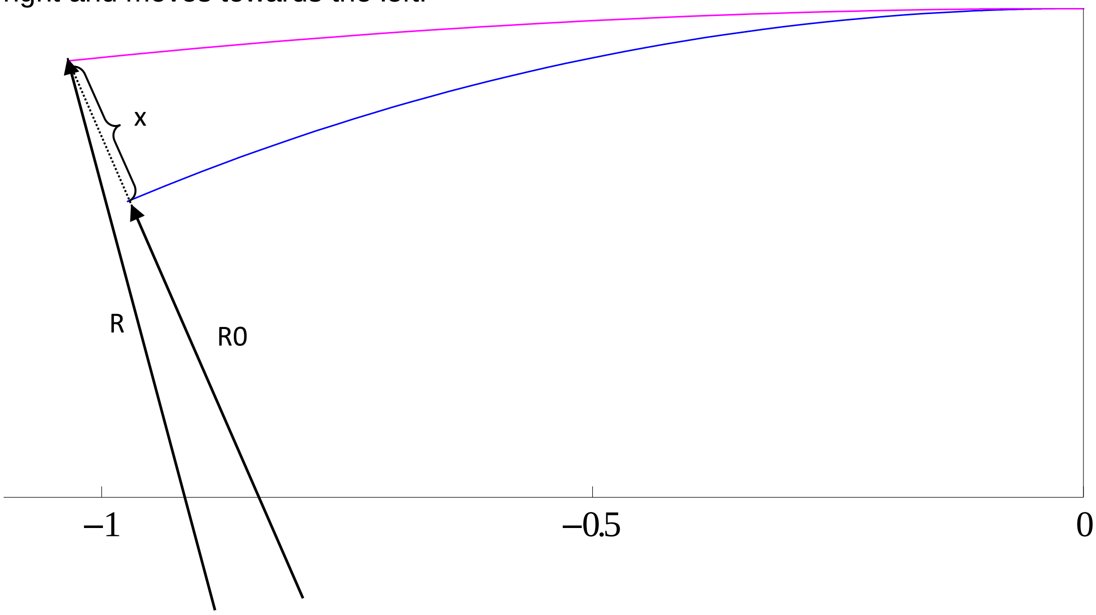


Figure 1. Trajectories of an off-momentum particle (rose) and a design energy particle(blue)

The angle of the off-momentum particle x' = 0.000395934. **This is the result from MADX!**
When it is converted to $\mathrm{px} = (1+\delta)\cdot \mathrm{x}'/\sqrt{1+\mathrm{x}'^2+\mathrm{y}'^2}$, where $\delta = 0.01$, then $\mathrm{px} = 0.000399893$, **which is the result from PTC**.

Example 1: Off-momentum beam:

Example with: $\delta = \frac{\Delta p}{p} = 0.09$ and relativistic $\beta \approx 1$:

```
dipole  : SBEND,     L=1, ANGLE=0.04;
```

	PTC		MADX
X0	0.00000000		0.00000000
PX0	0.00000000		0.00000000
T0	0.00000000		0.00000000
PT0	0.09000000		0.09000000
X1	0.00179976		0.00179976
PX1	0.00359904		0.00359904
T1	-0.00002400		-0.00002400
PT1	0.09000000		0.09000000
X2	-0.00016191		-0.00016191
PX2	0.00000000		0.00000000
T2	0.00000000		0.00000000
PT2	0.00000000		0.00000000
X1+X2	0.00163785		0.00163785
PX1+PX2	0.00359904		0.00359904
T1+T2	-0.00002400		-0.00002400
PT1+PT2	0.09000000		0.09000000
X	0.00165122		0.00165121
PX	0.00359904		0.00330187
T	0.00000000		0.00000016
PT	0.09000000		0.00000000

Table 13. PTC and MADX are equal for off-momentum beams - notice that for MADX, "px" must be multiplied with the factor $(1+\delta)$ - in this case `0.00330187 * (1+0.09) = 0.00359904`

Notice the following important observations:

- PT is zero for MADX, but PTC has the correct value for PT.
- The px variable from MADX needs to be multiplied with $(1+\delta)$ to give the same as PTC
- The results from MADX itself are not the same as the second order calculations, but are equal to the results from PTC i.e. the results from MADX are higher than 2'nd order.

Example 2: Off-momentum beam:

Example with: $\delta = \frac{\Delta p}{p} = 0.09$ and relativistic $\beta = 0.8$:

```
dipole   : SBEND,     L=1, ANGLE=0.04;
```

	PTC		MADX
X0	0.01000000		0.01000000
PX0	0.00000000		0.00000000
T0	0.00000000		0.00000000
PT0	0.07310241		0.07310241*
X1	0.01181932		0.01181932
PX1	0.00363815		0.00363815
T1	0.04058979		0.04058979
PT1	0.07310241		0.07310241
X2	-0.00019551		-0.00019551
PX2	-0.00006010		-0.00006010
T2	-0.00561826		-0.00561826
PT2	0.00000000		0.00000000
X1+X2	0.01162381		0.01162381
PX1+PX2	0.00357805		0.00357805
T1+T2	0.03497152		0.03497152
PT1+PT2	0.07310241		0.07310241
X	0.01164453		0.01164453
PX	0.00358304		0.00328720
T	0.00000000		-0.04218941
PT	0.07310241		0.00000000

* would have been 0.07200000 with the MADX/TWISS definition of the PT variable.

Table 13. PTC and MADX are equal for off-momentum beams - notice that for MADX, "px" must be multiplied with the factor $(1+\delta)$ - in this case `0.00328720* (1+0.09) = 0.00358305`

Notice the following important observations:

- PT is zero for MADX, but PTC has the correct value for PT.
- The px variable from MADX needs to be multiplied with $(1+\delta)$ to give the same as PTC
- The results from MADX itself are not the same as the second order calculations, but are equal to the results from PTC i.e. the results from MADX are higher than 2'nd order.

For off-momentum beams, with px not zero, the results from PTC and MADX are different

Example with: $\delta = \frac{\Delta p}{p} = 0.09$ and relativistic $\beta \approx 1$:

`dipole : SBEND, L=1, ANGLE=0.04;`

	PTC		MADX
X0	0.00000000		0.00000000
PX0	0.03270000		0.03000000[1]
T0	0.00000000		0.00000000
PT0	0.09000000		0.09000000
X1	0.03449104		0.03449104
PX1	0.03627288		0.03627288
T1	-0.00067791		-0.00067791
PT1	0.09000000		0.09000000
X2	-0.00309109		-0.00309109
PX2	-0.00002138		-0.00002138
T2	-0.00053450		-0.00053450
PT2	0.00000000		0.00000000
X1+X2	0.03139995		0.03139995
PX1+PX2	0.03625150		0.03625150
T1+T2	-0.00121241		-0.00121241
PT1+PT2	0.09000000		0.09000000
X	0.03167008		0.03165418
PX	0.03625326		0.03325988
T	0.00000000		-0.00109917
PT	0.09000000		0.00000000

[1] Since MADX use x' for PX, then 0.03 is used as input for MADX. 0.0327 is used in the matrix calculations.

Table 14. The “x” from PTC and MADX is different for off-momentum beams with “px” input - notice that for MADX, "px" must be multiplied with the factor $(1+\delta)$ - in this case `0.03325988` *(1+0.09)= `0.03625327`

Notice the following important observations:

- PT is zero for MADX, but PTC has the correct value for PT.
- The px variable from MADX needs to be multiplied with $(1+\delta)$ to give the same as PTC
- The results from MADX itself are not the same as the second order calculations, but are similar to the results from PTC i.e. the results from MADX are higher than 2'nd order.
- The “x” variable from MADX is different from the “x” from PTC

The difference of the “x” variable between PTC and MADX is also observed for small δ.

RBENDS are as SBENDs with pole face rotations, both for PTC and MADX, i.e. RBEND+K1 = SBEND + E1 + E2+K1,

It is always the case that an RBEND is modeled as an SBEND with pole face rotations on each end. Whether a dipole is calculated as SBEND+E1+E2+K1 or RBEND+K1, the transfer maps and the results stays the same.

Example with: $\delta = \frac{\Delta p}{p} = 0.01$ and relativistic $\beta = 0.99999987$:

dipole : RBEND, L=1, ANGLE=0.4, K1=0.1;
dipole : SBEND, L=1, ANGLE=0.4, E1=0.4/2, E2=0.4/2, K1=0.1;

	PTC		MADX
X0	0.01000000		0.01000000
PX0	0.03030000		0.03000000
T0	0.00000000		0.00000000
PT0	0.01000000		0.01000000
X	0.04040216		0.04039614
PX	0.03159165		0.03131696
T	0.00000000		-0.01035068
PT	0.01000000		0.00000000

Table 15. This table can be calculated with either a SBEND with pole face rotations or as a RBEND. In this example PTC and MADX give different results, because the beam is off-axis.

FRINGE FIELDS have only effects in the orthogonal plane. This is the case in both PTC and MADX

Fringe fields are specified with the parameters FINT (FINTX for differences between entry and exit), HGAP and H1 (H2 for the exit).

The fringe fields are only acting on the orthogonal parameters (see reference [11]). Therefore the fringe fields have no effect calculations, unless the orthogonal input parameters are different from zero.

In the following example, as in all the examples in this paper,
the vertical input parameters Y0 and PY0 are zero:

dipole : SBEND, L=1, ANGLE=0.04; FINT=0.5, HGAP=0.8

	No fringe fields		With fringe fields	
	PTC	**MADX**	**PTC**	**MADX**
X0	0.10000000	0.10000000	0.10000000	0.10000000
PX0	0.03000000	0.03000000	0.03000000	0.03000000
T0	0.00000000	0.00000000	0.00000000	0.00000000
PT0	0.00000000	0.00000000	0.00000000	0.00000000
X	0.13005403	0.13004056	0.13005403	0.13004056
PX	0.02979805	0.02979805	0.02979805	0.02979805
T	0.00000000	-0.00631092	0.00000000	-0.00631092
PT	0.00000000	0.00000000	0.00000000	0.00000000

Table 16. Adding the parameters for fringe fields to PTC or MADX, does not change the results (as long as Y0 and PY0 are zero)

Adding fringe fields, does not change the first order maps, neither from PTC, MADX nor TRANSPORT. With fringe fields, the matrix elements in second order MAPS from PTC and MADX are different for many elements. Also the second order maps from MADX and TRANSPORT are different, but only for 6 elements. The second order transfer maps for PTC, MADX and TRANSPORT for the above example is in appendix B.

We cannot make any conclusion whether PTC or MADX handles fringe fields best.

Conclusion:

The PX, PY, T and PT variables are defined differently for PTC and MADX. However, the first- and second order maps are the same for on-axis beams. This gives a contradiction in MADX, where the input parameters for MADX itself are given with MADX definitions, while the matrix calculations for MADX are done with PTC definitions of the input parameters. The conclusion is that all matrix calculations should be done with PTC definitions of the variables.

The first order maps for MADX are changing with the input parameters. Also the second order map change for off-momentum beams. These matrices only give meaningful results for on-axis input parameters. Therefore, all matrix calculations in this report are done with the on-axis matrices, as these are identical for PTC and MADX.

For off-momentum beams and with “px” input different from zero, then the “x” parameter is inaccurately calculated in MADX.

The results from MADX itself and the results from PTC itself are in many cases much more similar than MADX compared to the second order calculations.

PTC and MADX treat RBEND+K1 differently. MADX and TRANSPORT are identical for RBEND+K1.

PTC, MADX and TRANSPORT all treats fringe fields differently.

Acknowledgements:

E.Forrest and F.Schmidt; much good information, discussions and emails

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Appendix A Second order maps for PTC, MADX and TRANSPORT for a RBEND+K1

```
dipole  : RBEND,     L=1, ANGLE=0.04, K1=0.1;! beta=0.99999987, X0=0.1, PX0=0.03
```

PTC. Second order map:

"T111T611"	-0.00191797	-0.00009481	0.00000000	0.00000000	-0.00163399	0.00000000
"T121T621"	0.01804531	-0.00004779	0.00000000	0.00000000	0.02378424	0.00000000
"T131T631"	0.00000000	0.00000000	-0.00004997	-0.00409626	0.00000000	0.00000000
"T141T641"	0.00000000	0.00000000	0.02064689	-0.00195107	0.00000000	0.00000000
"T151T651"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T161T661"	0.02495934	-0.00078664	0.00000000	0.00000000	0.00032348	0.00000000
"T112T612"	0.01804531	-0.00004779	0.00000000	0.00000000	0.02378424	0.00000000
"T122T622"	0.00909616	-0.01903620	0.00000000	0.00000000	-0.48353351	0.00000000
"T132T632"	0.00000000	0.00000000	-0.01935156	-0.00206482	0.00000000	0.00000000
"T142T642"	0.00000000	0.00000000	0.01040755	0.01834289	0.00000000	0.00000000
"T152T652"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T162T662"	-0.48294117	0.02421137	0.00000000	0.00000000	0.00016306	0.00000000
"T113T613"	0.00000000	0.00000000	-0.00004997	-0.00409626	0.00000000	0.00000000
"T123T623"	0.00000000	0.00000000	-0.01935156	-0.00206482	0.00000000	0.00000000
"T133T633"	-0.00400144	-0.00410436	0.00000000	0.00000000	-0.00242004	0.00000000
"T143T643"	-0.02135219	-0.00211703	0.00000000	0.00000000	-0.02543082	0.00000000
"T153T653"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T163T663"	0.00000000	0.00000000	-0.02499206	-0.00000286	0.00000000	0.00000000
"T114T614"	0.00000000	0.00000000	0.02064689	-0.00195107	0.00000000	0.00000000
"T124T624"	0.00000000	0.00000000	0.01040755	0.01834289	0.00000000	0.00000000
"T134T634"	-0.02135219	-0.00211703	0.00000000	0.00000000	-0.02543082	0.00000000
"T144T644"	-0.03059951	-0.02172187	0.00000000	0.00000000	-0.51687060	0.00000000
"T154T654"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T164T664"	0.00000000	0.00000000	-0.51665295	-0.02423172	0.00000000	0.00000000
"T115T615"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T125T625"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T135T635"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T145T645"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T155T655"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T165T665"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T116T616"	0.02495934	-0.00078664	0.00000000	0.00000000	0.00032348	0.00000000
"T126T626"	-0.48294117	0.02421137	0.00000000	0.00000000	0.00016306	0.00000000
"T136T636"	0.00000000	0.00000000	-0.02499206	-0.00000286	0.00000000	0.00000000
"T146T646"	0.00000000	0.00000000	-0.51665295	-0.02423172	0.00000000	0.00000000
"T156T656"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T166T666"	-0.01965789	0.00064464	0.00000000	0.00000000	0.00000224	0.00000000

```
dipole  : RBEND,     L=1, ANGLE=0.04, K1=0.1;! beta=0.99999987, X0=0.1, PX0=0.03
```

MADX. Second order map:

"T111T611"	-0.00191889	-0.00009654	0.00000000	0.00000000	-0.00163398	0.00000000
"T121T621"	0.01804531	-0.00004866	0.00000000	0.00000000	0.02378424	0.00000000
"T131T631"	0.00000000	0.00000000	-0.00004983	-0.00409599	0.00000000	0.00000000
"T141T641"	0.00000000	0.00000000	0.02064689	-0.00195094	0.00000000	0.00000000
"T151T651"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T161T661"	0.02495934	-0.00078665	0.00000000	0.00000000	0.00032348	0.00000000
"T112T612"	0.01804531	-0.00004866	0.00000000	0.00000000	0.02378424	0.00000000
"T122T622"	0.00909616	-0.01903710	0.00000000	0.00000000	-0.48353351	0.00000000
"T132T632"	0.00000000	0.00000000	-0.01935156	-0.00206468	0.00000000	0.00000000
"T142T642"	0.00000000	0.00000000	0.01040755	0.01834302	0.00000000	0.00000000
"T152T652"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T162T662"	-0.48294117	0.02421135	0.00000000	0.00000000	0.00016306	0.00000000
"T113T613"	0.00000000	0.00000000	-0.00004983	-0.00409599	0.00000000	0.00000000
"T123T623"	0.00000000	0.00000000	-0.01935156	-0.00206468	0.00000000	0.00000000
"T133T633"	-0.00400130	-0.00410409	0.00000000	0.00000000	-0.00242005	0.00000000
"T143T643"	-0.02135219	-0.00211689	0.00000000	0.00000000	-0.02543082	0.00000000
"T153T653"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T163T663"	0.00000000	0.00000000	-0.02499206	-0.00000286	0.00000000	0.00000000
"T114T614"	0.00000000	0.00000000	0.02064689	-0.00195094	0.00000000	0.00000000
"T124T624"	0.00000000	0.00000000	0.01040755	0.01834302	0.00000000	0.00000000
"T134T634"	-0.02135219	-0.00211689	0.00000000	0.00000000	-0.02543082	0.00000000
"T144T644"	-0.03059951	-0.02172173	0.00000000	0.00000000	-0.51687060	0.00000000
"T154T654"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T164T664"	0.00000000	0.00000000	-0.51665295	-0.02423172	0.00000000	0.00000000
"T115T615"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T125T625"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T135T635"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T145T645"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T155T655"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T165T665"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T116T616"	0.02495934	-0.00078665	0.00000000	0.00000000	0.00032348	0.00000000
"T126T626"	-0.48294117	0.02421135	0.00000000	0.00000000	0.00016306	0.00000000
"T136T636"	0.00000000	0.00000000	-0.02499206	-0.00000286	0.00000000	0.00000000
"T146T646"	0.00000000	0.00000000	-0.51665295	-0.02423172	0.00000000	0.00000000
"T156T656"	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
"T166T666"	-0.01965789	0.00064464	0.00000000	0.00000000	0.00000224	0.00000000

```
BEAM     0.000      0      0.000      0       0.000      0     1 ;
B1 : SBEND, L=1., ANGLE= 0.04, E1=0.02, E2=0.02, K1=0.1 ;
```

TRANSPORT. Second order map:

```
1 11 -1.919E-03
1 12  3.609E-02  1 22  9.096E-03
1 13  0.000E+00  1 23  0.000E+00  1 33 -4.001E-03
1 14  0.000E+00  1 24  0.000E+00  1 34 -4.270E-02  1 44 -3.060E-02
1 15  0.000E+00  1 25  0.000E+00  1 35  0.000E+00  1 45  0.000E+00  1 55  0.000E+00
1 16  4.992E-02  1 26  1.727E-02  1 36  0.000E+00  1 46  0.000E+00  1 56  0.000E+00  1 66 -1.966E-02

2 11 -9.654E-05
2 12 -9.733E-05  2 22 -1.904E-02
2 13  0.000E+00  2 23  0.000E+00  2 33 -4.104E-03
2 14  0.000E+00  2 24  0.000E+00  2 34 -4.234E-03  2 44 -2.172E-02
2 15  0.000E+00  2 25  0.000E+00  2 35  0.000E+00  2 45  0.000E+00  2 55  0.000E+00
2 16  9.679E-02  2 26  4.842E-02  2 36  0.000E+00  2 46  0.000E+00  2 56  0.000E+00  2 66 -3.870E-02

3 11  0.000E+00
3 12  0.000E+00  3 22  0.000E+00
3 13 -9.967E-05  3 23 -3.870E-02  3 33  0.000E+00
3 14  4.129E-02  3 24  2.082E-02  3 34  0.000E+00  3 44  0.000E+00
3 15  0.000E+00  3 25  0.000E+00  3 35  0.000E+00  3 45  0.000E+00  3 55  0.000E+00
3 16  0.000E+00  3 26  0.000E+00  3 36 -4.998E-02  3 46 -1.656E-02  3 56  0.000E+00  3 66  0.000E+00

4 11  0.000E+00
4 12  0.000E+00  4 22  0.000E+00
4 13 -8.192E-03  4 23 -4.129E-03  4 33  0.000E+00
4 14 -3.902E-03  4 24  3.669E-02  4 34  0.000E+00  4 44  0.000E+00
4 15  0.000E+00  4 25  0.000E+00  4 35  0.000E+00  4 45  0.000E+00  4 55  0.000E+00
4 16  0.000E+00  4 26  0.000E+00  4 36 -1.000E-01  4 46 -4.846E-02  4 56  0.000E+00  4 66  0.000E+00

5 11 -1.634E-03
5 12  4.757E-02  5 22 -4.835E-01
5 13  0.000E+00  5 23  0.000E+00  5 33 -2.420E-03
5 14  0.000E+00  5 24  0.000E+00  5 34 -5.086E-02  5 44 -5.169E-01
5 15  0.000E+00  5 25  0.000E+00  5 35  0.000E+00  5 45  0.000E+00  5 55  0.000E+00
5 16  6.469E-04  5 26 -1.951E-02  5 36  0.000E+00  5 46  0.000E+00  5 56  0.000E+00  5 66  2.625E-06
```

Appendix B Second order maps for PTC, MADX and TRANSPORT for a SBEND with fringe fields.

```
dipole  : SBEND,     L=1, ANGLE=0.04, FINT=0.5, HGAP=0.8; ! beta~1, X0=0, PX0=0
```

PTC. Second order map:

```
"T111T611" -0.00003198  0.00000000  0.00000000  0.00000000 -0.00000000 0.0000000
"T121T621"  0.01997867  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T131T631"  0.00000000  0.00000000  0.00002560 -0.00003203  0.00000000 0.0000000
"T141T641"  0.00000000  0.00000000  0.01999467 -0.00000642  0.00000000 0.0000000
"T151T651"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T161T661"  0.00079957 -0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T112T612"  0.01997867  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T122T622"  0.00999067 -0.01999467  0.00000000  0.00000000 -0.49986668 0.0000000
"T132T632"  0.00000000  0.00000000 -0.02000769 -0.00001602  0.00000000 0.0000000
"T142T642"  0.00000000  0.00000000  0.00999867  0.02001728  0.00000000 0.0000000
"T152T652"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T162T662" -0.49946684  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T113T613"  0.00000000  0.00000000  0.00002560 -0.00003203  0.00000000 0.0000000
"T123T623"  0.00000000  0.00000000 -0.02000769 -0.00001602  0.00000000 0.0000000
"T133T633" -0.00006733 -0.00003206  0.00000000  0.00000000 -0.00208438 0.0000000
"T143T643" -0.02005893 -0.00002560  0.00000000  0.00000000 -0.00128152 0.0000000
"T153T653"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T163T663"  0.00000000  0.00000000  0.00000061  0.00208376  0.00000000 0.0000000
"T114T614"  0.00000000  0.00000000  0.01999467 -0.00000642  0.00000000 0.0000000
"T124T624"  0.00000000  0.00000000  0.00999867  0.02001728  0.00000000 0.0000000
"T134T634" -0.02005893 -0.00002560  0.00000000  0.00000000 -0.00128152 0.0000000
"T144T644" -0.03001916 -0.01999467  0.00000000  0.00000000 -0.50050733 0.0000000
"T154T654"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T164T664"  0.00000000  0.00000000 -0.49986668  0.00080121  0.00000000 0.0000000
"T115T615"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T125T625"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T135T635"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T145T645"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T155T655"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T165T665"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T116T616"  0.00079957 -0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T126T626" -0.49946684  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T136T636"  0.00000000  0.00000000  0.00000061  0.00208376  0.00000000 0.0000000
"T146T646"  0.00000000  0.00000000 -0.49986668  0.00080121  0.00000000 0.0000000
"T156T656"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T166T666" -0.01998934 -0.00000000  0.00000000  0.00000000 -0.00000000 0.0000000
```

```
dipole  : SBEND,     L=1, ANGLE=0.04, FINT=0.5, HGAP=0.8; ! beta~1, X0=0, PX0=0
```

MADX. Second order map:

```
"T111T611" -0.00003198  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T121T621"  0.01997867  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T131T631"  0.00000000  0.00000000  0.00002560 -0.00003200  0.00000000 0.0000000
"T141T641"  0.00000000  0.00000000  0.01999467 -0.00000639  0.00000000 0.0000000
"T151T651"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T161T661"  0.00079957  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T112T612"  0.01997867  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T122T622"  0.00999067 -0.01999467  0.00000000  0.00000000 -0.49986668 0.0000000
"T132T632"  0.00000000  0.00000000 -0.01998720 -0.00001600  0.00000000 0.0000000
"T142T642"  0.00000000  0.00000000  0.00999867  0.01999680  0.00000000 0.0000000
"T152T652"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T162T662" -0.49946684  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T113T613"  0.00000000  0.00000000  0.00002560 -0.00003200  0.00000000 0.0000000
"T123T623"  0.00000000  0.00000000 -0.01998720 -0.00001600  0.00000000 0.0000000
"T133T633" -0.00006726 -0.00003202  0.00000000  0.00000000 -0.00080061 0.0000000
"T143T643" -0.02003841 -0.00002560  0.00000000  0.00000000 -0.00064005 0.0000000
"T153T653"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T163T663"  0.00000000  0.00000000 -0.00064005  0.00079999  0.00000000 0.0000000
"T114T614"  0.00000000  0.00000000  0.01999467 -0.00000639  0.00000000 0.0000000
"T124T624"  0.00000000  0.00000000  0.00999867  0.01999680  0.00000000 0.0000000
"T134T634" -0.02003841 -0.00002560  0.00000000  0.00000000 -0.00064005 0.0000000
"T144T644" -0.02999867 -0.01999467  0.00000000  0.00000000 -0.49986668 0.0000000
"T154T654"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T164T664"  0.00000000  0.00000000 -0.49986668  0.00015974  0.00000000 0.0000000
"T115T615"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T125T625"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T135T635"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T145T645"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T155T655"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T165T665"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T116T616"  0.00079957  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T126T626" -0.49946684  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T136T636"  0.00000000  0.00000000 -0.00064005  0.00079999  0.00000000 0.0000000
"T146T646"  0.00000000  0.00000000 -0.49986668  0.00015974  0.00000000 0.0000000
"T156T656"  0.00000000  0.00000000  0.00000000  0.00000000  0.00000000 0.0000000
"T166T666" -0.01998934 -0.00000000  0.00000000  0.00000000 -0.00000000 0.0000000
```

```
UMAD
BEAM    0.000       0       0.000       0        0.000       0     1 ;
B1 : SBEND, L=1., ANGLE= 0.04, FINT=0.5, HGAP=0.8 ;
```

TRANSPORT. Second order map:

```
1 11 -3.198E-05
1 12  3.996E-02  1 22  9.991E-03
1 13  0.000E+00  1 23  0.000E+00  1 33 -6.725E-05
1 14  0.000E+00  1 24  0.000E+00  1 34 -4.008E-02  1 44 -3.000E-02
1 15  0.000E+00  1 25  0.000E+00  1 35  0.000E+00  1 45  0.000E+00  1 55  0.000E+00
1 16  1.599E-03  1 26  7.997E-04  1 36  0.000E+00  1 46  0.000E+00  1 56  0.000E+00  1 66 -1.999E-02

2 11 -1.095E-12
2 12  9.937E-13  2 22 -1.999E-02
2 13  0.000E+00  2 23  0.000E+00  2 33 -3.202E-05
2 14  0.000E+00  2 24  0.000E+00  2 34 -5.119E-05  2 44 -1.999E-02
2 15  0.000E+00  2 25  0.000E+00  2 35  0.000E+00  2 45  0.000E+00  2 55  0.000E+00
2 16  1.600E-03  2 26  5.045E-11  2 36  0.000E+00  2 46  0.000E+00  2 56  0.000E+00  2 66 -3.999E-02

3 11  0.000E+00
3 12  0.000E+00  3 22  0.000E+00
3 13  5.119E-05  3 23 -3.997E-02  3 33  0.000E+00
3 14  3.999E-02  3 24  2.000E-02  3 34  0.000E+00  3 44  0.000E+00
3 15  0.000E+00  3 25  0.000E+00  3 35  0.000E+00  3 45  0.000E+00  3 55  0.000E+00
3 16  0.000E+00  3 26  0.000E+00  3 36 -2.560E-03  3 46  2.666E-04  3 56  0.000E+00  3 66  0.000E+00

4 11  0.000E+00
4 12  0.000E+00  4 22  0.000E+00
4 13 -6.400E-05  4 23 -3.201E-05  4 33  0.000E+00
4 14 -1.280E-05  4 24  3.999E-02  4 34  0.000E+00  4 44  0.000E+00
4 15  0.000E+00  4 25  0.000E+00  4 35  0.000E+00  4 45  0.000E+00  4 55  0.000E+00
4 16  0.000E+00  4 26  0.000E+00  4 36 -3.525E-03  4 46 -9.601E-04  4 56  0.000E+00  4 66  0.000E+00

5 11  0.000E+00
5 12  0.000E+00  5 22 -4.999E-01
5 13  0.000E+00  5 23  0.000E+00  5 33 -8.006E-04
5 14  0.000E+00  5 24  0.000E+00  5 34 -1.280E-03  5 44 -4.999E-01
5 15  0.000E+00  5 25  0.000E+00  5 35  0.000E+00  5 45  0.000E+00  5 55  0.000E+00
5 16  0.000E+00  5 26 -2.000E-02  5 36  0.000E+00  5 46  0.000E+00  5 56  0.000E+00  5 66  0.000E+00
```

Appendix C First and second order maps for TRANSPORT for a quadrupole.

$$R11 = Cos[\sqrt{k} \cdot l]$$

$$R12 = \frac{1}{\sqrt{k}} \cdot Sin[\sqrt{k} \cdot l]$$

$$R21 = -\sqrt{k} \cdot Sin[\sqrt{k} \cdot l]$$

$$R22 = Cos[\sqrt{k} \cdot l]$$

$$R33 = Cosh[\sqrt{k} \cdot l]$$

$$R34 = \frac{1}{\sqrt{k}} \cdot Sinh[\sqrt{k} \cdot l]$$

$$R43 = -\sqrt{k} \cdot Sinh[\sqrt{k} \cdot l]$$

$$R44 = Cosh[\sqrt{k} \cdot l]$$

$$T116 = \tfrac{1}{2} \cdot \sqrt{k} \cdot l \cdot Sin[\sqrt{k} \cdot l]$$

$$T126 = \frac{1}{2}\left[\frac{1}{\sqrt{k}} \cdot Sin[\sqrt{k} \cdot l] - l \cdot Cos[\sqrt{k} \cdot l]\right]$$

$$T216 = \frac{\sqrt{k}}{2} \cdot \left[\sqrt{k} \cdot l \cdot Cos[\sqrt{k} \cdot l] + Sin[\sqrt{k} \cdot l]\right]$$

$$T226 = \tfrac{1}{2} \cdot \sqrt{k} \cdot l \cdot Sin[\sqrt{k} \cdot l]$$

$$T336 = -\tfrac{1}{2} \cdot \sqrt{k} \cdot l \cdot Sinh[\sqrt{k} \cdot l]$$

$$T346 = \frac{1}{2}\left[\frac{1}{\sqrt{k}} \cdot Sinh[\sqrt{k} \cdot l] - l \cdot Cosh[\sqrt{k} \cdot l]\right]$$

$$T436 = -\frac{\sqrt{k}}{2} \cdot \left[\sqrt{k} \cdot l \cdot Cosh[\sqrt{k} \cdot l] + Sinh[\sqrt{k} \cdot l]\right]$$

$$T446 = -\tfrac{1}{2} \cdot \sqrt{k} \cdot l \cdot Sinh[\sqrt{k} \cdot l]$$

Calculation for a quadrupole, up to 7'th order:

http://cern-accelerators-optics.web.cern.ch/cern-accelerators-optics/OtherInfo/Quadrupole_MADX.xlsx

Exact calculation for a quadrupole:

http://cern-accelerators-optics.web.cern.ch/cern-accelerators-optics/OtherInfo/Quadrupole_MADX.nb