Resistivity of a sinusoidally corrugated surface

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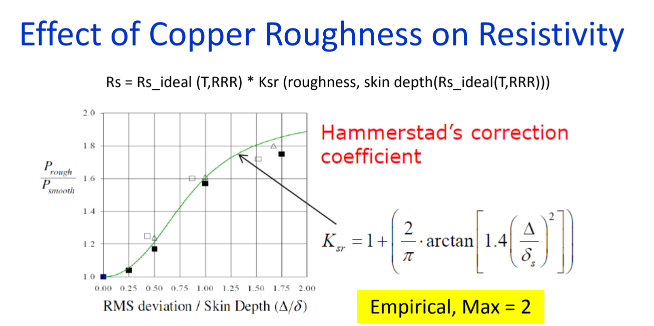
Introduction

From ChrisA's talk. Paper by S. Morgan, Journal of Applied Physics 20, 352 (1949).



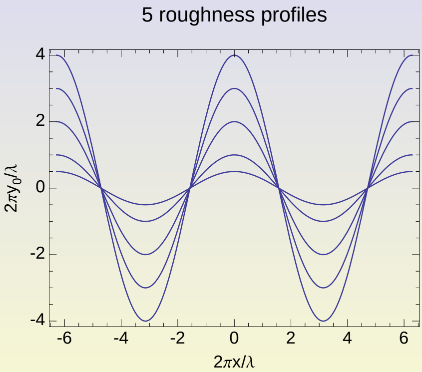
There is an important parameter missing in this plot: corrugation slope $\sim \Delta/\lambda$.

Model

Assume the sin profile of the surface

$$y_0(x) = h \cos\left(\frac{2\pi x}{\lambda}\right)$$

These are grooves parallel to z-axis.



For $2\pi h/\lambda = 4$ the steepest slope angle is 76°.

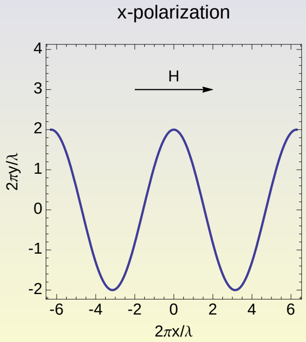
Direction of the magnetic field

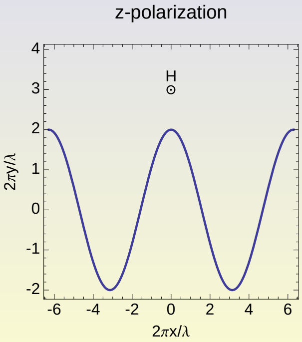
Magnetic field can be considered uniform at distance from the surface $\gg$ corrugation scale (but much smaller than the RF wavelength). There are two independent directions (polarizations) of the magnetic field. Far from the surface the magnetic field approaches a constant value

$$\boldsymbol{H} = \hat{\boldsymbol{x}} H_0$$

or

$$\boldsymbol{H} = \hat{\boldsymbol{z}} H_0$$





Small skin depth limit

Assume small skin depth, $\delta \ll h, \lambda/2\pi$. The boundary condition for the magnetic field on the surface is

$$H_n|_{\text{surface}} = 0$$

Resistivity increase factor

$$\eta = \frac{\int_{\text{period}} H_t^2 \, ds}{H_0^2 \lambda}$$

We will calculate η for both polarizations, η_x, η_z. If magnetic field is randomly oriented relative to the grooves, averaging over all possible orientations gives

$$\eta = \frac{1}{2}(\eta_x + \eta_y)$$

For the z-polarization the magnetic field penetrates the grooves, $\boldsymbol{H}(x, y) = \hat{\boldsymbol{z}} H_0$ for $y > y_0(x)$, and

$$\eta = \frac{1}{\lambda} \int_{\text{period}} ds$$

Small skin depth limit - x polarization

For x-polarization one has to solve the Poisson equation above the metal, $y > y_0(x)$:

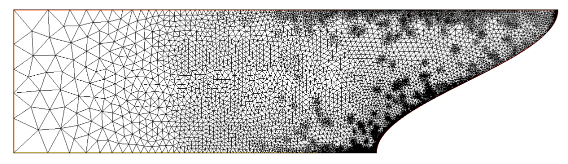
$$\boldsymbol{H}(x, y) = -\hat{\boldsymbol{z}} \times \nabla\psi(x, y), \qquad \Delta\psi = 0$$

with the boundary conditions

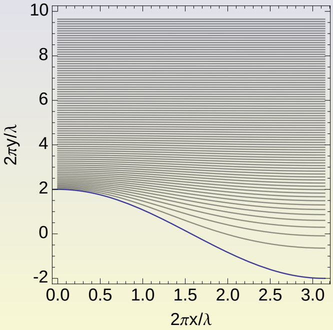
$$\psi_{\rm surf} = {\rm const}, \qquad \psi|_{y\to\infty} \to H_0 y$$

The function ψ is periodic along x with the period λ.
I used computer code FreeFem++ (`http://www.freefem.org/ff++/index.htm`) to numerically solve the Poisson equation.

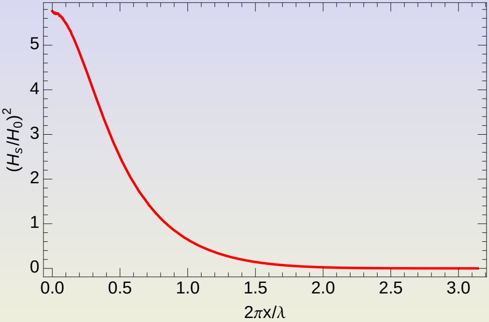
Case $2\pi h/\lambda = 2$



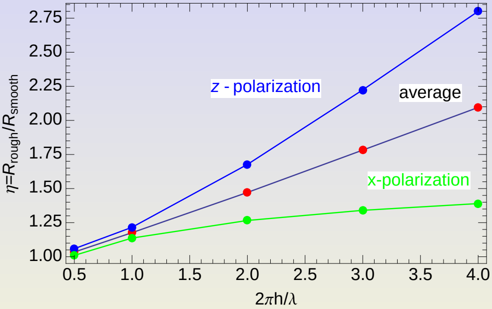
Field lines



Magnetic field on the surface



Increase of resistivity



How to solve 3D case in the limit of small skin depth?

3D geometry metal surface

$$y = h(x, z)$$

Magnetic field in free space

$$\boldsymbol{H}(x, y, z) = \nabla\psi(x, y, z), \qquad \Delta\psi = 0$$

Boundary condition on the surface

$$\left.\frac{\partial\psi}{\partial n}\right|_{y=h(x,z)} = 0$$

Boundary condition at infinity, $\boldsymbol{H} \to \hat{\boldsymbol{x}} H_0$

$$\psi|_{y\to\infty} \to H_0 x$$

Solve the Poisson equation in a large $0 < x < a$, $0 < z < b$ area (say, with periodic boundary conditions) and compute $\int_{\mathrm{surf}} H_t^2 dS / H_0^2 ab$.

Finite skin depth case, z-polarization

For the z-polarization the magnetic field penetrates the grooves, $\boldsymbol{H}(x, y) = \hat{\boldsymbol{z}} H_0$ for $y > y_0(x)$. In the metal

$$\Delta H_z = \frac{2i}{\delta^2} H_z$$

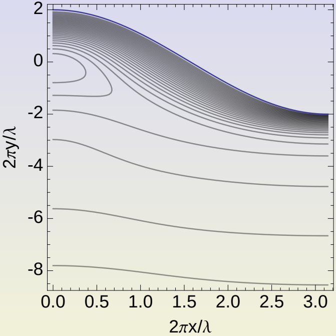
From Morgan's paper

$$\eta = \frac{2}{H_0 \lambda \delta} \mathrm{Im} \int_{\mathrm{period}} dx \int_{-\infty}^{y_0(x)} H_z \, dy$$

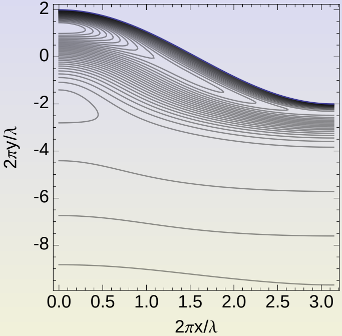
Finite skin depth case, z-polarization

Case $2\pi h/\lambda = 2$, $2\pi\delta/\lambda = 0.6$

Contour lines of Re H_z

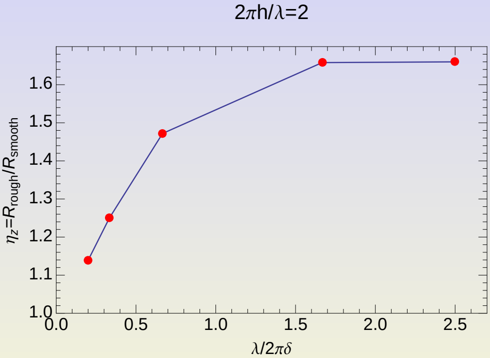


Contour lines of Im H_z



Finite skin depth case, z-polarization

This is the case studied in Morgan's paper (for different profiles of grooves).



Note that $\lambda/2\pi\delta = h/2\delta$. In the limit $\delta \to 0$ we previously found $\eta_z = 1.68$.

Finite skin depth case, x-polarization

Take

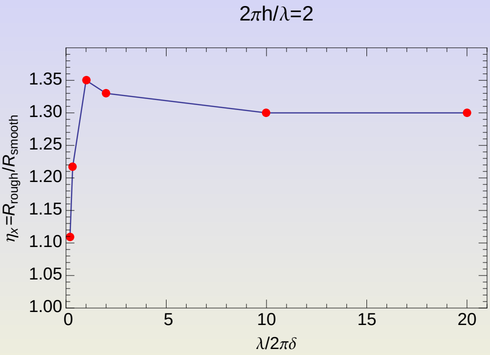
$$\boldsymbol{H} = -\hat{\boldsymbol{z}} \times \nabla \psi(x, y)$$

At $y \to \infty$ we have $\psi \to H_0 y$, and at $y \to -\infty$ we have $\psi \to 0$.
The equation for ψ

$$\Delta \psi = s \frac{2i}{\delta^2} \psi$$

where the function $s = 1$ in the metal and zero otherwise.

Finite skin depth case, x-polarization



In the limit $\delta \to 0$ we previously found $\eta_x = 1.27$